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TESTING THE QUANTITY-QUALITY FERTILITY MODEL:

RESULTS FROM A NATURAL EXPERIMENT

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The proposition that the quantity and quality of children interact because parents tend to want approximately equal levels of quality for each of their children (Becker, 1960; Becker and Lewis, 1973; Willis, 1973; Becker and Tomes, 1976) has provided a unifying rationale for why, among other phenomena, rises in income tend to reduce fertility and quality and numbers of children tend to be negatively correlated across households within a country and across societies.[1] The hypothesis has also provided a linkage between the household-theoretic approach to fertility and other frameworks (Easterlin, 1968). The essence of the interaction model is that the shadow price of quality per child (Q) depends on the number (N) of children while the shadow price of quantity depends on the level of quality chosen. A rise in full income which alters the levels of N and Q will therefore also induce price effects so that the relationship between income and N or Q, holding observable market prices constant, may differ from the 'true' income effect (shadow prices held constant).

Because of the central role of the unobservable shadow prices, the question naturally arises whether the interaction hypothesis is capable of being refuted with existing or any data. In section I, we present the model and compare its comparative statics properties to those of the standard non-interactive consumer model. It is demonstrated that the minimal set of restrictions necessary to "identify" an interaction between Q and N requires estimates of compensated price effects involving prices of N and Q which are not dependent on commodity levels. In section II we utilize rationing theory to show how a "natural" experiment present in almost all large micro data sets, the occurence of multiple births, can serve as a substitute for variation in these prices and in section III utilize household data from India to test the theory using the "twins" under alternative restrictions on the utility function. As a by-product of the twins experiment, estimates of the directional impact of an exogenous change in fertility of schooling per child and household expenditures on durables are obtained which are immune to identification problems and robust to omitted variables bias.

I. The Basic Model, Fixed Prices, and Reduced Form Estimates

In the basic three-commodity interaction (q^2) model of Becker and Lewis, the household maximizes a utility function (1) with numbers of children, quality per child and a composite commodity S as arguments.

$$(1) \qquad U = U\ (N,Q,S)$$

The full income constraint of the household is

$$(2) \qquad F = NQ\Pi + p_N N + p_Q Q + \Pi_S S$$

where F = full income, Π is the price-weighted sum of the cost-minimizing levels of quality inputs required to augment the quality of one child by one unit, Π_S is a similar shadow price for the composite commodity, and p_N and p_S are 'fixed' prices such that $p_N N$ is the component of total child costs that is independent of the level of Q chosen and $p_Q Q$ is that part of child costs which is independent of the level of fertility. We comment on the empirical counterparts to these prices below. Both Π and Π_S are assumed to be invariant with respect to the levels of the commodities produced.

Note that this model is a special case of the general quantity-quality model of Theil (1952) and Houthakker (1952), where every commodity has both a quality and quantity component, and that the conventional three-commodity (q^1) model is a special case of the fixed price q^2 model in which $\Pi = 0$.[2]

Maximization of (1) subject to (2) yields the first-order conditions

(3) $$U_N = \lambda[\Pi Q + p_N] = \lambda\Pi_N$$

(4) $$U_Q = \lambda[\Pi N + p_Q] = \lambda\Pi_Q$$

(5) $$U_S = \lambda\Pi_S$$

where λ is the marginal utility of full income. Π_N and Π_Q are thus the respective shadow prices of N and Q, which can be seen to depend in part on the levels of Q and N chosen.

The existence of the fixed prices allows the derivation of compensated own and cross substitution effects which are analogous to those of the conventional three-commodity consumer demand model. Letting the determinant of the bordered Hessian matrix of the model in (1) and (2) be given by Δ and the co-factor from the ith row and jth column of the Hessian matrix from the consumer demand model in which there is no interaction be ϕ_{ij}, we can write the compensated substitution effects in terms of the fixed prices as:[3]

(6) $$\left(\frac{\delta N}{\delta p_N}\right)_{\bar{U}} = \frac{\lambda\phi_{NN}}{\Delta} = \alpha_1 < 0$$

(7) $$\left(\frac{\delta Q}{\delta p_Q}\right)_{\bar{U}} = \frac{\lambda\phi_{QQ}}{\Delta} = \beta_2 < 0$$

(8) $$\left(\frac{\delta S}{\delta \Pi_S}\right)_{\bar{U}} = \frac{\lambda\phi_{SS}}{\Delta} - \frac{2\lambda^2\Pi\Pi_Q\Pi_N}{\Delta} = \gamma_3 < 0$$

(9) $$\left(\frac{\delta Q}{\delta p_N}\right)_{\bar{U}} = \left(\frac{\delta N}{\delta p_Q}\right)_{\bar{U}} = \frac{\lambda\phi_{NQ}}{\Delta} - \frac{\lambda^2\Pi\Pi_S^2}{\Delta} = \beta_1 = \alpha_2 \lesseqgtr 0$$

(10) $$\left(\frac{\delta N}{\delta \Pi_S}\right)\bar{U} = \left(\frac{\delta S}{\delta p_N}\right)\bar{U} = \frac{\lambda\phi_{NS}}{\Delta} + \frac{\lambda^2 \Pi\Pi_S\Pi_Q}{\Delta} = \alpha_3 = \gamma_1 \lesseqgtr 0$$

(11) $$\left(\frac{\delta Q}{\delta \Pi_S}\right)\bar{U} = \left(\frac{\delta S}{\delta p_Q}\right) = \frac{\lambda\phi_{QS}}{\Delta} + \frac{\lambda^2 \Pi\Pi_N\Pi_S}{\Delta} = \beta_3 = \gamma_2 \lesseqgtr 0$$

where the α_i, β_i and γ_i are coefficients from the set of reduced-form commodity demand equations containing full income;

(12) $$N = \alpha_0 + \alpha_1 p_N + \alpha_2 p_Q + \alpha_3 \Pi_S + \alpha_4 F + \sum_{j=5}^{M} \alpha_j Z_j + \varepsilon_1$$

(13) $$Q = \beta_0 + \beta_1 p_N + \beta_2 p_Q + \beta_3 \Pi_S + \beta_4 F + \sum_{j=5}^{M} \beta_j Z_j + \varepsilon_2$$

(14) $$S = \gamma_0 + \gamma_1 p_N + \gamma_2 p_Q + \gamma_3 \Pi_S + \gamma_4 F + \sum_{j=5}^{M} \gamma_j Z_j + \varepsilon_3$$

the Z_j are exogenous prices hypothesized to enter into Π and the ε_i are random error terms.[4]

Given the availability of information on all fixed prices and F, it is easy to see that estimation of (12), (13), and (14) without further prior information on the characteristics of the utility function does not allow one to discern whether there is an interaction between Q and N or whether the true underlying model is one in which Π=0, i.e., the q^1 model. Note that this does not mean that one would therefore accept the conventional fixed price model (q^1) since the assumption that average quality is one of the three commodities would appear to require an interactive budget constraint. Thus, rejection of the q^2 model would likely necessitate a reformulation of the quantity-quality choice.

From (6) and (7) it is evident that the numerators of the compensated own price effects on N and Q are identical to those in the q^1 model and reflect the properties of the utility function only. The own effect for S and each of the observed or estimated compensated cross price effects, however, contain an additional term which is constrained by second-order conditions to be negative, since $\Delta<0$.[5] It is these additional terms containing Π which are the essence of the q^2 model; with $\Pi=0$, the compensated price effects evidently collapse to those of the conventional framework. The q^2 model thus adds one more (unobservable) unknown, Π, but does not add an additional, linearly independent equation. Neither the cofactors nor the Π terms can therefore be identified even if all reduced-form price or income effects can be estimated, as pointed out by Theil (1952).[6]

Moreover, the symmetry conditions that hold for the compensated price effects of the conventional model hold for the observed (fixed) price effects of the q^2 model and it can also be readily demonstrated that the usual adding-up constraints that hold for the ϕ_{ij} also hold for the observed compensated price effects, i.e.,

$$(15) \qquad \sum_j \left(\frac{\delta i}{\delta \Pi_j}\right) \Pi_j = \sum_j \phi_{ij}\Pi_j = 0 \quad i,j = N, Q, S$$

Subject to (15), none of the cross effects are signed and own effects have the same (negative) sign as in the q^1 model. Unless the ϕ_{ij} are restricted or are known, the signs or magnitudes of the observed compensated price effects and/or income effects do not provide any evidence of the quantity-quality interactions, that is, of the existence of the hypothesized Π terms. Observable phenomena which can be "explained" by the q^2 model can thus also be accounted for by the non-interactive q^1 framework.

It is easy to show, however, that the imposition of a linearly independent set of restrictions on any cofactors related to compensated price effects in the absence of an interaction between N and Q (obviously, excluding the combination composed solely of ϕ_{QQ} and ϕ_{NN}) or a restriction on true (unobserved) income effects is sufficient to compute the values of all the cofactors, given the availability of estimates of the own fixed price effects for Q and N and the average values of the shadow prices Π_Q, Π_N, and Π_S.[7] The signs of the Π terms can then be inferred from the reduced form estimates and expressions (8) through (11). It is not necessary, however, to compute all the cofactors and/or all the shadow prices (or budget shares) to obtain a refutable prediction from the model when fixed price effects are known. As examples, consider three alternative linear combinations of cofactor pairs $\phi_{NQ} = \rho_1\phi_{QS}$, $\phi_{NQ} = \rho_2\phi_{NS}$, $\phi_{QS} = \rho_3\phi_{NS}$. These restrictions imply that:

$$(16) \qquad 1 - \rho_1 \frac{\hat{\beta}_3 \quad (=\hat{\gamma}_2)}{\hat{\beta}_1 \quad (=\hat{\alpha}_2)} = -\frac{\lambda^2 \Pi\Pi_S[\Pi_S+\rho_1\Pi_N]}{\Delta\hat{\beta}_1}$$

$$(17) \qquad 1 - \rho_2 \frac{\hat{\alpha}_3 \quad (=\hat{\gamma}_1)}{\hat{\beta}_1 \quad (=\hat{\alpha}_2)} = -\frac{\lambda^2 \Pi\Pi_S[\Pi_S+\rho_2\Pi_Q]}{\Delta\hat{\beta}_1}$$

$$(18) \qquad 1 - \rho_2 \frac{\hat{\alpha}_3 \quad (=\hat{\gamma}_1)}{\hat{\beta}_3 \quad (=\hat{\alpha}_2)} = -\frac{\lambda^2 \Pi\Pi_S[\Pi_N-\rho_3\Pi_Q]}{\Delta\hat{\beta}_3}$$

If it is assumed that $\rho_1 \geq 0$ so that both N and S must be substitutes for Q, i.e. ϕ_{NQ}, $\phi_{QS} < 0$, then the R.H.S. of (16) must be positive; that is, relative to the (assumed) substitutibility of N and Q with S implied by the utility function, the quantity-quality interaction implies that the effect of an increase in the fixed price $p_Q(p_N)$ will be less negative on N(Q) than on S. One need only then specify a prior upper bound on ρ_1 to establish a refutable prediction of the quantity-quality model without the need to compute shadow prices. Mutatis mutandis, an upper bound for ρ_2 could be established. If the L.H.S. of (17) became positive only at levels of ρ_2 above this prior bound, the theory would be rejected. Similar conclusions obtain for ρ_3; if it is a strongly held prior that, say, S is a substitute for N but is complementary with Q ($\rho_3 \leq 0$), then the R.H.S. of (18) is signed. Establishment of a lower bound for ρ_3 sets the parameter of the test. If the acceptable prior for the range of any ρ_i contains negative and positive values, however, then the relative magnitudes of the shadow prices Π_S, Π_N and Π_Q must also be known to obtain refutable hypotheses from (16), (17), and (18).

If none of the fixed prices vary in a given data set, even more restrictive assumptions about the characteristics of the utility function are required to identify the interaction model. The compensated effects on the three commodities of a change in a Z_j hypothesized to affect Π can be written as:

$$(19) \qquad \frac{\partial N}{\partial Z_j} = \left(\frac{\partial N}{\partial \Pi}\right)_{\bar{U}} \frac{\partial \Pi}{\partial Z_j} = \left[\frac{\lambda\phi_{NN}Q}{\Delta} - \frac{\lambda\phi_{NQ}N}{\Delta} + \frac{\lambda^2\Pi\Pi_S{}^2N}{\Delta}\right]\frac{\partial \Pi}{\partial Z_j} \gtrless 0$$

$$(20) \qquad \frac{\partial Q}{\partial Z_j} = \left(\frac{\partial Q}{\partial \Pi}\right)_{\bar{U}} \frac{\partial \Pi}{\partial Z_j} = \left[\frac{\lambda\phi_{QQ}N}{\Delta} - \frac{\lambda\phi_{NQ}Q}{\Delta} + \frac{\lambda^2\Pi\Pi_S{}^2Q}{\Delta}\right]\frac{\partial \Pi}{\partial Z_j} \lesseqgtr 0$$

$$(21) \qquad \frac{\partial S}{\partial Z_j} = \left(\frac{\partial S}{\partial \Pi}\right)_{\bar{U}} \frac{\partial \Pi}{\partial Z_j} = \left[\frac{\lambda\phi_{NS}Q}{\Delta} + \frac{\lambda\phi_{QS}N}{\Delta} + \frac{\lambda\Pi\Pi_S}{\Delta}(\Pi_Q Q+\Pi_N N)\right]\frac{\partial \Pi}{\partial Z_j} \gtrless 0$$

As can be seen, these shadow price effects are linear combinations of fixed price effects and are unsigned, although as Theil (1952) has shown the sum of compensated own quality and quantity shadow price elasticities must be negative. Note, however, that $\frac{\partial Q}{\partial Z_j}$ or $\frac{\partial N}{\partial Z_j}$ could be positive if the Z_j affect the <u>fixed</u> prices of both N and Q simultaneously without an interaction. Thus, without knowledge of the own fixed price effects <u>and</u> some restriction on the ϕ_{ij}, $j \neq i$, the sign (existence) of the extra term containing Π cannot be ascertained from the Z_j effects. The hypothesis that the Z_j jointly affect the fixed prices of Q and N in the same direction cannot be distinguished from the hypothesis that $\Pi > 0$ and Q and N interact.

II. A 'Natural' Experiment and Constrained Demand Estimates

A practical problem with the tests which require the weakest prior restrictions on the utility function is that even one fixed price is unlikely to be available from most data sets, or, if measurable, is unlikely to vary across sample observations.[8] For instance, contraceptive costs, a likely candidate for p_N, are difficult to quantify and are not likely to vary in a cross-section. The availability of a price of quality per child which is not dependent on the quantity of children is even less likely, yet as we have shown, variations in these variables are critical to the test of the theory. Without fixed price variability, the test of the q^2 model would be restricted to a comparison of the compensated effects of Π_S on N and Q, as given by (18). Thus if priors on the substitution relationship between Q and S relative to N and S were diffuse, no test of the q^2 model would be possible.

Nature has, however, evidently provided an experiment whereby "extra" children are distributed randomly among households of given parity in many societies.[9] To the extent that multiple births from one pregnancy are unanticipated and children cannot readily be sold, households with "twins" can be considered to have experienced an exogenous increase in N above the level which would otherwise have been acquired. To see this, assume that planned family size N^* and Q take into account contraceptive costs and biological constraints on child spacing and, for simplicity, that there is no foetal wasteage or child mortality. 'Twins' clearly augment N above N^* and reduce welfare for women who experience the multiple birth at pregnancy N^*. Unanticipated multiple births at or below $N^* - k$, where k is the birth multiple, also raise levels of N above N^*

because the period over which contraception must be practiced lengthens, thus raising contraceptive costs (p_N). Put another way, women who experience multiple births while young bear greater risks of exceeding planned fertility and experience a welfare loss if their optimal birth intervals exceed the biological minimum. Households for whom the biological constraint on spacing is binding, however, experience a welfare gain from twins in the $N^* - k$ pregnancy which is an offset to the welfare loss due to the rise in contraceptive costs. The empirical implications of the parity-twin relationships are discussed more fully in section III below.

We now show how estimates of the effects of twins on the consumption of other household commodities, including the quality of the non-twin children, can be used to test the quality-quantity theory when p_N and/or p_Q do not vary in the sample, as long as the investigator is willing to impose at least one restriction on the utility function cofactor relationships. The appropriate "twins" model is similar to that given by (1) and (2) except that N is fixed at $\bar{N}$. The constrained choice set thus contains only Q and S and first order conditions are:

(22) $$U_Q = \lambda[\Pi\bar{N} + p_Q]$$

(23) $$U_S = \lambda\Pi_S$$

(24) $$F - \Pi\bar{N}Q - p_N\bar{N} - p_QQ - \Pi_SS = 0$$

Treating $\bar{N}$ as a parameter, we can totally differentiate expressions (22) through (24) to obtain the effect of an exogenous increase in $\bar{N}$ on Q and S, the "twins" effects. It can readily be shown that at the optimum (unconstrained) level of N (N^*), these effects can be written as:

(25) $$\frac{\delta Q}{\delta \bar{N}} = \left[\phi_{NQ} - \lambda \Pi \Pi_S^2\right] / \phi_{NN} = \left(\frac{\delta Q}{\delta p_N}\right)_{\bar{U}} \cdot \left(\frac{\delta N}{\delta p_N}\right)_{\bar{U}}^{-1} = \frac{\beta_1}{\alpha_1} = \frac{\alpha_2}{\alpha_1} \lesseqgtr 0$$

(26) $$\frac{\delta S}{\delta \bar{N}} = \left[\phi_{NS} + \lambda \Pi \Pi_S \Pi_Q\right] / \phi_{NN} = \left(\frac{\delta S}{\delta p_N}\right)_{\bar{U}} \cdot \left(\frac{\delta N}{\delta p_N}\right)_{\bar{U}}^{-1} = \frac{\gamma_1}{\alpha_1} = \frac{\alpha_3}{\alpha_1} \gtreqless 0$$

The effect of twins on quality per child is thus the <u>compensated</u> cross price effect of p_N on Q divided by the <u>compensated</u> own price effect on N from the unconstrained model, while the twins effect on S is equal to the ratio of the unconstrained compensated cross fixed price effect $(\partial S/\partial p_N)_{\bar{U}}$ to the own price effect on N. Note that in the usual three-commodity model ($\Pi = 0$) (25) and (26) would conform to the well-known result from rationing theory (Tobin and Houthakker, 1951) whereby the direction of the change in the consumption of good i induced by a change in the (non-marketable) rationed good j indicates whether i and j are complements or substitutes. In this case, because of the extra terms resulting from the non-linear budget constraint, the signs of the observed twins effects may not provide any information on whether N and Q or N and S are true substitutes or complements, i.e., the sign of ϕ_{NQ} and ϕ_{NS}. Indeed, as indicated by the expressions for the observed compensated cross price effects, the positive relationship between N and the shadow price of Q does not necessarily imply that an exogenous increase in N will reduce quality per child, since Q and N may be (strong) complements. Thus the sign of (25) (or (26)) is not sufficient information to test the theory; again a further restriction must be imposed. However, the greater the own price effect of N and the less the cross price effects of p_N on Q and S, the less the impact of multiple births on the consumption of child quality and S.

Estimates of the two twins effects on Q and S combined with estimates of the compensated cross effects of Π_S on N and Q from reduced forms are, however, sufficient to test the theory given linear restrictions on either of the paired cofactors ϕ_{NS} and ϕ_{NQ} or ϕ_{QS} and ϕ_{NQ}, as the twins "experiment" substitutes for variation in the fixed price p_N. To see this, note that the twins effect on Q (25) divided by the twins effect on S (26) provides the test given by (17) for a specified prior range for ρ_2. Alternatively, estimates of α_3 and β_3 combined with the twins effects yield (with ρ_1 specified) test (16) since $\left(\frac{\partial Q}{\partial \bar{N}} \Big/ \frac{\partial S}{\partial \bar{N}}\right) \cdot \frac{\alpha_3}{\beta_3} = \frac{\beta_2}{\beta_3}$.[10]

III. Results from the "Twins" Experiment: India

In this section we make use of the twins experiment in a national sample of 2,939 farm households in India to obtain information on the range of restrictions characterizing the substitution relationships between household commodities which are consistent with the interaction model. The estimates also provide information on the exogenous impact of a fertility change on the demand for schooling and other household goods. The data set employed, the Additional Rural Income Survey, collected in three rounds between 1969 and 1971 by the National Council of Applied Economic Research in New Delhi, was selected because 1) the sample is large enough to yield a significant number of twins, 2) pregnancy rosters for all married women are provided, enabling the identification of twins households and the birth order of multiple births, 3) average family size is sufficiently high so that the effects of multiple births on investment in non-twin children can be ascertained, and 4) levels of schooling investment are sufficiently low so that there are significant interfamily differences in the 'quality' of children remaining in the household.

The data provide information on the schooling attainment of every household member as well as on consumption expenditures. To measure the impact of twins on average child quality two age-standardized indices of schooling attainment, given by (27) at (28), were constructed,

$$\text{(27)} \qquad EDT = \frac{1}{N}\sum_{i=1}^{N} \frac{e_{ix}}{\bar{e}_x}$$

$$\text{(28)} \qquad ED = \frac{1}{NT}\sum_{i=1}^{NT} \frac{e_{ix}}{\bar{e}_x}$$

where N = number of children aged 5-14, NT = number of non-twins children 5-14, e_{ix} = schooling attainment of child i aged x and $\bar{e}_x$ = mean schooling attainment of all children aged x in the total sample.[11] The first index measures the average investment in schooling per child in the household relative to that in the total population, given the age composition of the children. The second index, which excludes the twins, is also employed because the inherent capabilities of twins may be, on average, lower than that of their siblings.[12] If parents do not fully compensate, inclusion of the schooling levels of the twins in the quality index will result in a spurious negative correlation between the presence of twins and average quality per child.[13] Comparison of the levels as well as the impact of multiple births on EDT and ED will thus shed light on whether or not parents fully compensate for lower endowments by higher investment.

As a proxy for the non-child household commodity S, we use a three-year average of expenditures on consumer durables. A shortcoming of the data set, however, is that no information on goods prices is provided so that the tests described by (16) and (18) cannot be performed. Moreover, to fix a bound for ρ_2 in test (17) and to obtain information on the relative substitutibility-complementarity between N and Q and N and S, an average price of durable goods must be computed to convert expenditures to quantities. This "price" was obtained by assuming that households purchase at most one durable good per year. The mean expenditures on durables for households with non-zero consumer durable expenditures, approximately 25 percent of households, is thus the average durable goods price, approximately 150 rupees.

Of the 2,939 households, 44 contained women who had had a multiple birth.[14] The restriction of the sample to households containing at least one non-twin child aged 5 to 14 and with information on consumption expenditures reduced the total to 1,644, 25 of which were "twins" households. Since the per-household probability of twins must be positively correlated with the number of pregnancies, given constant per-pregnancy risk, a variable merely representing the presence of twins would reflect desired fertility differences as well as unanticipated additions to family size. Moreover, as was discussed, the presence of twins has a differential impact depending on birth order. Use of the ratio of number of twins to number of pregnancies both standardizes for the risk of a multiple birth and captures birth order effects. Given that twins are random with respect to birth order, among women of the same age, those with more total pregnancies are less likely to have experienced a multiple birth at the last pregnancy. Thus, the smaller the ratio of twins to pregnancies, the smaller the impact of the twins effect, although, as we have argued above, there will still be twins effects if twins are produced at inframarginal pregnancies.

To estimate the twins effects on ED (EDT) and S, we employed regressions of the form:[15]

TABLE I

VARIABLE DEFINITIONS, MEANS AND STANDARD DEVIATIONS

TOTAL AND TWINS SAMPLES

Variable	Definition	Total Mean	Total S.D.	Twins Mean	Twins S.D.
EDT	Age-standardized schooling index including twins[a]	.97	.79	.46	.65
ED	Age-standardized schooling index excluding twins[a]	.97	.79	.48	.69
S	Three-year average of consumer durable expenditures (rupees)	47.70	127.9	27.40	39.40
TP	Number of multiple births per pregnancy	.003	.03	.20	.07
AGE	Age of mother	40.37	10.42	36.96	10.86
SEXRT	Female children/total children 5-14 including twins	.46	.36	.32	.34
SEXR	Female children/total children 5-14 excluding twins	.46	.36	.33	.41
B	Multiple birth in first pregnancy (=1)	.002	.049	.02	.05
D1	Death of one twin by date of survey (=1)	.0024	.049	.16	.37
D2	Death of both twins by date of survey (=1)	.0012	.035	.08	.28
n		1644		25	

[a] See text.

$$(29)\qquad ED(T) = \omega_0 + \omega_1 TP + \omega_2 AGE + \omega_3 SEXR(T) + \omega_4 B\cdot TP + \omega_5 D1\cdot TP + \omega_6 D2\cdot TP + U_1$$

$$(30)\qquad S = \theta_o + \theta_1 TP + \theta_2 AGE + \theta_3 SEXR(T) + \theta_4 B\cdot TP + \theta_5 D1\cdot TP + \theta_6 D2\cdot TP + U_2$$

All variables are defined in Table I, which also provides means and standard deviations. The interaction variables were included to test if (i) the arrival of twins at lower parities and (ii) the deaths of of one (D1=1) or both twins (D2=1) reduce the magnitude of the twins effects.[16] To standardize for the age of the mother (degree of completeness of family size) and the sex composition of the children including and excluding the twins, we also include AGE and SEXR(T) respectively.[17] Because the probability of a multiple birth, TP, is assumed to be random with respect to socioeconomic characteristics, however, the exclusion of other determinants of ED and S from (29) and (30) should not affect any of the twins coefficients.[18] The randomness of twins minimizes the possibility of bias due to omitted variables.

Table II reports the coefficient estimates from three specifications for each of the three dependent variables. The TP coefficients in the schooling equations indicate that an exogenous increase in family size due to the presence of twins reduces both the average educational attainment of all the children and that of the non-twin children in the household, with the negative effects on the non-twins being slightly less strong. The twins effects on Q appear to be measured relatively precisely and indicate that parents do not fully compensate for the presumed inherent 'inferiority' of twins.[19]

The inclusion of the other control variables does not appear to significantly alter the magnitudes of the estimated twins coefficients.

TABLE II

REGRESSION COEFFICIENTS: TWINS EFFECTS, TOTAL SAMPLE

(standard errors in parentheses)

Independent Variable	EDT (1)	EDT (2)	EDT (3)	ED (1)	ED (2)	ED (3)	S (1)	S (2)	S (3)
Constant	.980 (.020)	1.136 (.081)	1.136 (.081)	.981 (.020)	1.135 (.081)	1.135 (.081)	48.05 (3.20)	19.69 (13.33)	19.79 (13.36)
TP	-2.483 (.740)	-2.744 (.733)	-2.198 (.940)	-2.383 (.740)	-2.653 (.734)	-2.072 (.941)	-111.0 (120.3)	-92.02 (120.6)	-108.2 (154.6)
AGE		.0001 (.002)	.0001 (.002)		.0001 (.002)	.0001 (.002)		.667 (.305)	.665 (.306)
SEXRT		-.339 (.053)	-.339 (.053)					2.973 (8.665)	2.920 (8.678)
SEXR					-.337 (.053)	-.338 (.053)			
B•TP			.067 (2.801)			.274 (2.850)			115.6 (468.0)
D1•TP			-1.414 (1.675)			-1.600 (1.676)			28.91 (275.7)
D2•TP			-2.561 (3.085)			-2.719 (3.088)			-2.762 (507.62)
R^2	.007	.032	.032	.006	.031	.032	.001	.004	.004
F	11.28	17.61	8.99	10.37	17.22	8.86	.85	1.92	.97

The birth order interaction effects are in the expected direction, indicating that higher-order twins have a stronger (negative) impact on educational investment, but the mortality interaction coefficients display signs which are contrary to expectations. The negative signs of the latter may reflect, however, a positive correlation between household investments in health and in schooling, but the sets of interactions do not add significantly to the explanatory power of either schooling equation at conventional levels of significance.

The consumer durables results are less strong, with none of the explanatory variables explaining individually or collectively a significant proportion of the variance in S. The point estimates for the twins coefficients suggests, however, that the presence of an extra child reduces expenditures on consumer durables, with the effect again less negative when the multiple births occur at the first pregnancy.

In terms of the tests of the q^2 model, the negative twins effects in both the schooling and consumer durables equations indicate, from (9), (10), (25), and (26), that S and the quantity of children may be either substitutes or complements.[20] If both Q and S are assumed to be substitutes for N, the only assumption consistent with both the q^1 and q^2 models given our results, then, from (17), the critical value of ρ_2 which would lead to a rejection of the interaction model is 7.7. In elasticity terms this implies that the q^2 model would be rejected if the cross price elasticity between p_N and Q, defined in the conventional way, is as little as two percent greater than that between p_N and S.[21] This result, thus, does not rule out the case in which the (Allen) partial elasticities of substitution for all commodities are equal, one of the set of assumptions employed in Becker and Lewis, as long as the share of S in full income exceeds that of Q. The assumption that Q and N are complements, of course, would lead to a rejection of the q^1 model based on the negative twins effect on Q.

IV. Conclusions

In this paper we have demonstrated that refutable predictions cannot be derived from the quantity-quality model of fertility without the imposition of some structure on the household utility function. We also have shown that estimates of the relationships between quantity-independent (fixed) prices and commodity levels can be used to infer the existence of the unobservable interdependent shadow prices of the model with the least restrictive assumptions about the utility function and have shown how a unique, but ubiquitous, natural experiment, the occurrence of multiple births, can be used in place of variations in these fixed prices. Estimates of twins effects on average child schooling attainment and the consumption of consumer durables based on a sample of rural Indian farm households indicate that a plausible set of restrictions are ruled out by the interaction model. Regardless of priors on the validity of the restrictions, the twins experiment provides policy-relevant estimates of the sign of the impact of fertility on investment in schooling and in durables which are robust to omitted variables bias and which require no identification restrictions. The results obtained thus are the first to strongly confirm the hypothesis that exogenous reductions in fertility increase child quality and suggest that a decrease in family size brought about, say, by exogenous improvements in birth control technology, would increase schooling levels of Indian children.[22]

Footnotes

1. Quality is defined in this framework as the service flows per child which provide direct utility to the household. There is assumed to be an underlying production function for quality which relates inputs, such as the time of the family members and market goods, to the level of quality.

2. Theil derives the comparative statics results of the general quantity-quality model without fixed prices. Houthakker inserts fixed prices but ignores the comparative statics implications.

3. See Appendix.

4. These are assumed to be linear approximations of the true reduced form demand equations.

5. In the general quantity-quality model in which each good has a quality component, none of the own price effects exactly identify their corresponding cofactors.

6. Although income effects are not shown, it is easy to verify that, analogous to compensated price effects, each income effect contains an additional term involving Π. See Appendix.

7. See Appendix.

8. Restrictions on true income effects still require information on reduced form fixed price effects to distinguish the two models (see Appendix).

9. The randomness of multiple births may be attenuated in societies in which fertility drugs known to induce multiple births are used.

10. Note that test (18) is not independent of tests (16) and (17) because the prior values of the ρ's must be consistent with the adding up constraints.

11. Use of this measure in a linear regression context implies that the proportional change in schooling level induced by a unit change in an explanatory variable is independent of the age of the child. For a more

complete discussion of this standardization procedure. (See Rosenzweig, 1978)

12. The biological evidence indicates that the birthweight and gestation period of twins is substantially lower than that of non-twins, with multiple birth deliveries significantly more complicated (Dunn, 1965). Most of the evidence on ability and achievement differences is consistent with twins inferiority, but distinctions between inherent and investment differences are somewhat blurred (Mittler, 1971).

13. See Becker and Tomes (1976) for a discussion of the theoretical implications of compensation within the household.

14. Given the mean number of pregnancies for women of 4.2, the per-pregnancy probability of a multiple birth in the sample is .0036. This figure is significantly lower than the per birth probability in Great Britain (.0125) but is comparable to that in Japan (.0063) taking into account the significantly higher levels of foetal wastage in India (Gedda, 1961).

15. The coefficient ω_1 ($\omega_4=\omega_5=\omega_6=0$) is to be interpreted as the birth order impact of twins on average schooling, a non-twins family being equivalent to a twins family with an infinitely large number of pregnancies. As TP rises, i.e., as the number of pregnancies fall, the likelihood of the twins occurring at the final pregnancy increases. If twins occur last, only a fraction of the families will have overshot desired fertility, with the rest experiencing a rise in contraceptive costs. If the number of pregnancies is given by P and k_P is the fraction of families experiencing P pregnancies, the proportion of "overshooting" families is $(k_P/(k_P+k_{P+1}))\cdot(P-1)^{-1}$. Therefore, the approximate effect of an increase in TP on 'excess' fertility can be shown to be given by

$$\frac{\Delta(N-N^*)}{\Delta(1/P)} = \left[\frac{(1/(P''-1))(k_{P''}/(k_{P''}+k_{P''+1}))-(1/(P'-1))(k_{P'}/(k_{P'}+k_{P'+1}))}{(1/P'')-(1/P')}\right]$$

$$\times\left[1-\frac{\Delta(N-N^*)}{\Delta P_N}\cdot\frac{\Delta P_N}{\Delta T}\right]$$

where P' and P" are any two pregnancy levels and $\Delta(N-N^*)\Delta P_N \cdot \Delta P_N/\Delta T$ is the effect of the presence of twins on excess fertility via the contraceptive effect.

It should be noted that the second perenthetical term must be positive, since the occurrence of twins cannot increase N by more than the one additional child. Therefore, the sign of ω_1 must be the same as the sign of $\partial ED/\partial(N-N^*)$ but its magnitude will in general be different. Since the test of the quantity-quality model involves the ratio $\omega_1/\theta_1 = (\partial ED/\partial(N-N^*) \cdot \partial(N-N^*)/\partial(\frac{T}{P}))/(\partial S/\partial(N-N^*) \cdot \partial(N-N^*)/\partial(\frac{T}{P})) = (\partial ED/\partial(N-N^*)/(\partial S/\partial(N-N^*))$, information on $\partial(N-N^*)/\partial(\frac{T}{P})$ is unnecessary.

16. Deaths of twins at an early age would reduce their impact. However, the data do not indicate the age of death of any child. Moreover, deaths of children may be negatively correlated with overall child investment.

17. There is evidence that mother's age and the probability of a multiple birth are positively correlated (Mittler, 1971). Interaction of age with the twins variables led to results identical to those reported below.

18. Zero-order correlations between TP and the schooling levels of both parents, non-earnings income and farm size were all less than .05. Regression of TP on the complete set of socio-economic variables indicated acceptance of the null hypothesis of joint insignificance.

19. Direct comparisons of the mean achievement or schooling levels of twins to those of the total population do not, by themselves, provide evidence on the comparative capabilities of twins. Our results show that these differences have two components: the direct negative family size effect on the schooling level of every child in the twins household and the lower endowments of twins.

20. Note that adding up constraints (15) imply that both models would be rejected if ω_1 and θ_1 were positive.

21. The rejection criteria from (17) is

$$1 - \rho_2 \frac{\hat{\alpha}_3}{\hat{\beta}_1} < 0 .$$

Thus the rejection criterion for ρ_2 is

$$\rho_2 > \frac{\hat{\beta}_1}{\hat{\alpha}_3}$$

given that $\hat{\beta}_1 > 0$.

In terms of our regression coefficients

$$\frac{\hat{\beta}_1}{\hat{\alpha}_3} = \frac{\omega_1 \bar{Q}}{\theta_1 / \Pi_S} = \frac{(-2.38)(2.4)}{(-111)/150} = 7.7 ,$$

where $\bar{Q}$ is the mean years of schooling for 14 year olds and Π_S equals 150 rupees. The elasticities ratio is thus given by

$$\frac{\omega_1}{\theta_1} \Pi_S \bar{S} = \frac{(-2.38)(47.7)}{(-111)} = 1.02$$

and is invariant to the assumption underlying the estimate of Π_S, as $\Pi_S \bar{S}$ is the mean expenditure on durables.

22. There are a number of other important applications of the twins experiment involving the impact of exogenous changes in fertility on such phenomena as female labor force participation, savings, and marital stability. If contraceptive costs of twins can be isolated, based on contraceptive use data, then quantitative estimates can be obtained. The authors are currently working on these topics.

Appendix

Total differentiation of the first-order conditions (3)-(5) and the budget constraint (2) yields the following simultaneous equations system written in matrix form.

$$\text{A.1} \quad \begin{pmatrix} U_{NN} & U_{NQ}-\lambda\Pi & U_{NS} & -(P_N+Q\Pi) \\ U_{QN-\lambda\Pi} & U_{QQ} & U_{QS} & -(P_Q+N\Pi) \\ U_{SN} & U_{SQ} & U_{SS} & -\Pi_S \\ -(P_N+Q\Pi) & -(P_Q+N\Pi) & -\Pi_S & 0 \end{pmatrix} \begin{pmatrix} dN \\ dQ \\ dS \\ d\lambda \end{pmatrix} = \begin{pmatrix} \lambda dP_N+\lambda Qd\Pi \\ \lambda dP_Q+\lambda Nd\Pi \\ \lambda d\Pi_S \\ NQd\Pi+NdP_N+QdP_Q-dF \end{pmatrix}$$

In the q^1 model Π=$d\Pi$=0. Letting ϕ_{ij} (i,j=N,Q,S) be the ijth cofactor of the q^1 Hessian and Δ the determinant of the q^2 Hessian, equations (6)-(11) are easily derived. The income effects are given by

$$\text{A.2} \quad \alpha_4 = \frac{dN}{dF} = \frac{-\phi_{41} - \lambda\Pi\ (-\Pi_Q U_{NS} + \Pi_N U_{QS})}{\Delta}$$

$$\text{A.3} \quad \beta_4 = \frac{dQ}{dF} = \frac{-\phi_{42} - \lambda\Pi\ (-\Pi_S U_{NS} + \Pi_N U_{SS})}{\Delta}$$

$$\text{A.4} \quad \gamma_4 = \frac{dS}{dF} = \frac{-\phi_{43} + \lambda\Pi \left[-\Pi_S(U_{NQ}-\lambda\Pi) + \Pi_N U_{SQ}\right]}{\Delta}$$

It is obvious that utility function restrictions are required to identify the sign of Π.

Now consider the following "structural" equation set from which the "true" income effects a_4, b_4 and c_4 are derived. R is "total expenditures" on N, Q and S ($R = \Pi_N N + \Pi_Q Q + \Pi_S S$).

$$\text{A.5} \quad N = a_1\Pi_N + a_2\Pi_Q + a_3\Pi_S + a_4 R$$

$$\text{A.6} \quad Q = b_1\Pi_N + b_2\Pi_Q + b_3\Pi_S + b_4 R$$

A.7 $$S = c_1\Pi_N + c_2\Pi_Q + c_3\Pi_S + c_4R$$

A.8 $$\Pi_N = P_N + \Pi Q$$

A.9 $$\Pi_Q = P_Q + \Pi N$$

Then, reduced form income effects (holding P_N, P_Q, Π_S constant) are, in terms of structural parameters,

A.10 $$\alpha_4 = \frac{dN}{dF} = \frac{a_4(1-b_1\Pi)+a_1b_4\Pi}{(1-a_2\Pi)(1-b_1\Pi)-a_1b_1\Pi^2}$$

A.11 $$\beta_4 = \frac{dQ}{dF} = \frac{b_4(1-a_2\Pi)+b_1a_4\Pi}{(1-a_2\Pi)(1-b_1\Pi)-a_1b_1\Pi^2}$$

A.12 $$\gamma_4 = \frac{dS}{dF} = c_4 + c_1\Pi\frac{dN}{dF} + c_2\Pi\frac{dQ}{dF}$$

Note that observed income effects are weighted averages of "true" income effects (Becker and Lewis).

These structural parameters are themselves not estimable since Π_N and Π_Q are unobserved. But, all structural parameters may be expressed in terms of the reduced form parameters. Substitution in A.10 - A.12 for a_1, a_2, b_1, b_2, c_1, and c_2 in terms of their reduced form counterparts, e.g., $a_1 = \alpha_1 - \Pi\alpha_2$, leads to 3 equations and 4 unknowns, the three true income effects and Π. Any linear restriction on the true income effects given knowledge of the <u>fixed</u> price reduced form effects is sufficient to estimate Π and the true income effects. Conversely, prior information about true income effects is not sufficient to identify the sign of Π

without information about fixed prices.

For the constrained maximization (twins) problem, the demand equations (evaluated at the unconstrained optimum) for Q, S and λ in terms of $\bar{N}$ are given by

$$\text{A.13} \quad \begin{pmatrix} U_{QQ} & U_{QS} & -(P_Q+\Pi\bar{N}) \\ U_{SQ} & U_{SS} & -\Pi_S \\ -(P_Q+\Pi\bar{N}) & \Pi_S & 0 \end{pmatrix} \begin{pmatrix} \frac{dQ}{d\bar{N}} \\ \frac{dS}{d\bar{N}} \\ \frac{d\lambda}{d\bar{N}} \end{pmatrix} = \begin{pmatrix} -\lambda\Pi-U_{QN} \\ -U_{SN} \\ \Pi Q+P_N-\frac{dF}{d\bar{N}} \end{pmatrix}$$

Letting ϕ^c_{ij} (i,j = Q,S) be the ijth cofactor of the constrained Hessian and Δ^c its determinant, the following uncompensated twins effects are easily derived:

$$\text{A.14} \qquad \frac{dQ}{d\bar{N}} = \frac{-(\lambda\Pi+U_{QN})\ \phi^c_{QQ} - U_{SN}\phi^c_{SQ} + (\Pi Q+P_N)\phi^c_{33}}{\Delta^c}$$

$$\text{A.15} \qquad \frac{dS}{d\bar{N}} = \frac{-(\lambda\Pi+U_{QN})\phi^c_{QS} - U_{SN}\phi^c_{SS} + (\Pi Q+P_N)\phi^c_{32}}{\Delta^c}$$

From A.1, it is easily shown that (evaluated at the unconstrained optimum) $\Delta^c = \phi_{NN}$ and that the numerators of A.14 and A.15 are $\phi_{QN} - \lambda\Pi\Pi_S{}^2$ and $\phi_{SN} + \lambda\Pi\Pi_S\Pi_Q$ respectively.

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